GASDYNAMICS OF A PLASMA IN A MULTIPLE-MIRROR MAGNETIC TRAP WITH "POINT" MIRRORS

Yu. V. Vasil'ev and V. V. Mirnov

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Equations are derived for the gasdynamics of a dense plasma confined by a multiple-mirror magnetic field. The limiting cases of large and small mean free paths have been analyzed earlier: $\lambda \ll l_0$ and $\lambda \gg lk$, where l is the length of an individual mirror machine, l_0 is the size of the mirror, and k is the mirror ratio. The present work is devoted to a study of the intermediate range of mean free paths $l_0 \ll \lambda \ll lk$. It is shown that in this region of the parameters the process of expansion of the plasma has a diffusional nature, and the coefficients of transfer of the plasma along the magnetic field are calculated.

The possibility of the longitudinal thermal insulation of a plasma in open systems using a corrugated (multiple-mirror) magnetic field has been under discussion lately. The interest in this means of confinement is connected with the successes of experiments [1-3] which have been performed to verify the predictions of the theory [4-7]. One of the results of the theory consists in the fact that when a large mirror ratio $k \equiv H_{max}/H_{min} \gg 1$ is specified the optimum mode of thermal insulation is achieved in a multiple-mirror trap with "point" mirrors. Mirrors whose length l_0 is much less than the corrugation period l of the field are conventionally called point mirrors in [5, 6].

In addition to the most efficient confinement, the point mirrors are advantageous from the aspect of energy consumption required to create the magnetic field. This is important for a plasma with parameters close to thermonuclear

$$n \approx 10^{18}\ \mathrm{cm}^{-3} \qquad T \approx 5\ \mathrm{keV} \tag{1}$$

when large magnetic fields and mirror ratios are needed for good thermal insulation. The theory of plasma confinement by a corrugated field presented in [5, 6] pertain to the two opposite limiting cases: the case of purely Knudsen flow, which is realized with large mean free paths $\lambda \gg lk$, and the mode when the flow is hydrodynamic everywhere along a field tube ($\lambda \ll l_0$). Now if, having in mind the region of parameters (1), one takes $l_0 \approx 5$ cm and $l \approx 50$ cm for the estimates of the actual dimensions, then it turns out that as the plasma is heated a state is rapidly established where the flow has a Knudsen nature only at the mirrors, whereas in the other regions it has a hydrodynamic nature. This intermediate range of mean free paths

$$l_0 \ll \lambda \ll lk \tag{2}$$

was not studied in [5, 6].

The magnetic field confining the plasma will be taken as assigned. In the configuration of the field lines it represents a multiple-mirror trap consisting of a large number of mirror machines joined end to end. Since in the range of mean free paths $l_0 \ll \lambda$ the dynamics of the plasma expansion should not depend on the size and shape of a mirror, we will solve the problem in the null approximation with respect to the parameter l_0/λ, assuming that $l_0 = 0$ and the field in the space between mirrors is uniform.

The procedure for the derivation of the gasdynamic equations describing the dispersion along a field tube of a plasma cluster with a characteristic longitudinal scale L is analo-

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gous to the system of calculations used in [5, 6]. First one must solve the steady-state problem and relate the fluxes of matter q_α and energy Q_α of the two plasma components having the drops in concentrations Δn_α and temperatures ΔT_α (the subscript denotes the sort of particles, α = i, e) and the potential $\Delta\varphi$ between the centers of two adjacent mirror machines. At this stage of the calculations one must assume that the differences Δn_α, ΔT_α, and $\Delta\varphi$ are assigned and do not depend on time.

For determinacy we introduce the coordinate s along the field tube in such a way that a mirror separating two mirror machines is located at the point s = 0. Then

$$\begin{aligned} \Delta n_a &= n_a(l/2) - n_a(-l/2) \\ \Delta T_a &= T_a(l/2) - T_a(-l/2) \\ \Delta\varphi &= \varphi(l/2) - \varphi(-l/2) \end{aligned} \tag{3}$$

As in [5, 6], we will assume that the plasma cluster is sufficiently extended and occupies a large number of mirror machines ($N \approx L/l \gg 1$). In this case the values of n_α, T_α, and φ vary slightly in the scale of a single mirror machine ($\Delta n_\alpha \ll n_\alpha$, $\Delta T_\alpha \ll T_\alpha$). The nature of their variation is considerably different in different sections of a field tube: within the mirror machines where the flow has a collisional nature the parameters n_α, T_α, and φ vary smoothly in accordance with the equations of two-fluid hydrodynamics. In the region of a mirror the hydrodynamic approximation breaks down. Here the plasma parameters can undergo sharp jumps whose spatial scale is on the order of the size of the mirror. Therefore, in a solution of the problem in a null approximation with respect to l_0/λ one must take into account the fact that the boundary values of n_α, T_α, and φ may not coincide to the left and to the right of the mirror.

We can determine the jumps in these values upon the transition through a mirror by the equalities

$$\begin{aligned} &\delta n_a = n_a(+0) - n_a(-0),\ \ \delta T_a = T_a(+0) - T_a(-0) \\ &\delta\varphi = \varphi(+0) - \varphi(-0) \end{aligned} \tag{4}$$

For the further calculations we will use the assumption that the mirror ratio is large ($k \gg 1$). This assumption allows us to find in explicit form the connection between the fluxes of matter and energy, on the one hand, and the values (4), on the other. Let us examine the flow of any component of the plasma through a mirror. If the mirror ratio were equal to infinity, then the exchange of particles between plasma machines would be absent and a Maxwellian velocity distribution with parameters corresponding to the given mirror machine would be established within each of them. With a finite mirror ratio the Maxwellian distribution near a mirror is disturbed by the flux of particles from the next mirror machine which has different values of the parameters (concentration, temperature, and potential). The disturbance in the distribution function is proportional to the difference in the values of the parameters and also, since the mirror ratio is large, to the smallness of the number of particles $\sim n/k$ penetrating from mirror machine to mirror machine

$$f(\mathrm{v}) - f_M(\mathrm{v}) = \delta f(\mathrm{v}) \sim \frac{1}{k}\frac{\Delta n}{n}$$

In the calculation of the fluxes of matter and energy the contribution to the integral from the function $f(v)$ gives a narrow region of phase space $v_\perp^2/v^2 = \sin^2\theta \le 1/k$. Keeping this in mind, one can verify that the allowance for a disturbance $\delta f(v)$ would lead to the appearance of terms containing the small value k^{-1} in the equations for the fluxes. Therefore, the fluxes of matter and energy can be calculated with an accuracy of the terms of order k^{-1} by assuming that the distribution functions near a mirror are Maxwellian. The results of these calculations performed in a linear approximation with respect to δn_α, δT_α, and δ have the form

$$\begin{aligned} q_a &= -\frac{n_a}{k}\left(\frac{T_a}{2\pi m_a}\right)^{1/2}\left(\frac{\delta n_a}{n_a} + \frac{1}{2}\frac{\delta T_a}{T_a} \pm \frac{e\delta\varphi}{T_a}\right) \\ Q_a &= -\frac{n_a T_a}{k}\left(\frac{T_a}{2\pi m_a}\right)^{1/2}\left(2\frac{\delta n_a}{n_a} + 3\frac{\delta T_a}{T_a} \pm 2\frac{e\delta\varphi}{T_a}\right) \end{aligned} \tag{5}$$

In the mode when the plasma cluster expands freely the condition of quasineutrality is expressed by the equality $q_e = q_i \equiv q$, which allows one to eliminate from (5) the jump in potential at a mirror. Equations (5) take the form

$$
\begin{aligned}
q &= -\frac{1}{k(2\pi m_i T_i)^{1/2}}\left[\left(T_e + T_i\right)\delta n + \frac{n}{2}(\delta T_e + \delta T_i)\right]\\
Q_e &= -\frac{n}{k}\left(\frac{2T_e}{\pi m_e}\right)^{1/2}\delta T_e\\
Q_i &= -\frac{1}{k}\left(\frac{T_i}{2\pi m_i}\right)^{1/2}[2(T_e+T_i)\delta n + 3n\delta T_i + n\delta T_e]
\end{aligned}
\tag{6}
$$

Terms containing the small value m_e/m_i are omitted in Eqs. (6).

Now we can find the connections between the jumps at a mirror, which figure in (6), and the assigned drops in the parameters (3) between adjacent mirror machines. The transition from free flow at the mirrors to hydrodynamic flow in the central section of a mirror machine takes place near a mirror at distances on the order of the effective mean free path. We can take the effective mean free path as the distance $\lambda_{eff} \approx \lambda/k$ in which a flying particle is scattered through an angle $\Delta^2\theta \approx k^{-1}$ and enters the region of phase space occupied by trapped particles. If $\lambda_{eff} \ll l$, and the inequality (2) guarantees that this condition is satisfied, then the equations of two-fluid hydrodynamics can be used everywhere within the mirror machines to describe the plasma flow [8].

We can join the steady-state solutions for the regions of Knudsen and hydrodynamic flows by using the fact that the fluxes of matter and energy are constant along a field tube with good accuracy. The variations in these values along the length of a mirror machine are quantities of the second order of smallness with respect to the parameter l/L, whereas the fluxes are values of the first order of smallness. By equating the hydrodynamic equations for the fluxes of matter and energy with the values of (6) found we obtain equations describing the temperature distributions in the hydrodynamic region:

$$
-\varkappa_a \frac{\partial T_a}{\partial s} + \frac{5}{2} q T_a = Q_a \tag{7}
$$

The thermal-conductivity coefficients are given by the equations

$$
\varkappa_i = 1.63\frac{T_i^{5/2}}{\Lambda e^4 m_i^{1/2}}, \quad \varkappa_e = 0.93\frac{T_e^{5/2}}{\Lambda e^4 m_e^{1/2}}
$$

These equations must be supplemented by the condition of steadiness of the flow. Neglecting viscosity, it comes down to the requirement of constancy of the total pressure along a field tube

$$
\frac{\partial}{\partial s} n(T_e + T_i) = 0 \tag{8}
$$

We integrate Eqs. (7) and (8) with respect to s between the centers of the mirror machines (from the point $s = -l/2$ to $s = l/2$), excluding a region of width $\delta(\lambda_{eff} \ll \delta \ll l)$ on both sides of the mirror where the hydrodynamic approximation breaks down. As a result we obtain

$$
\begin{aligned}
&\frac{\varkappa_a}{l}[\Delta T_a - T_a(\delta) + T_a(-\delta)] = Q_a - \frac{5}{2} q T_a\\
&n(\Delta T_e + \Delta T_i) + (T_e + T_i)\Delta n = (T_e + T_i)[n(\delta) - n(-\delta)] +\\
&+ n[T_e(\delta) - T_e(-\delta)] + n[T_i(\delta) - T_i(-\delta)]
\end{aligned}
\tag{9}
$$

Since $\delta \ll l$, the differences $T_a(\delta) - T_a(-\delta)$ and $n(\delta) - n(-\delta)$ are equal to the jumps (4) in concentration and temperatures at the mirror with the accuracy of terms of order δ/l. With allowance for this we can rewrite (9) in the form

$$
\begin{aligned}
&\frac{\varkappa_a}{l}(\Delta T_a - \delta T_a) = Q_a - \frac{5}{2} q T_a\\
&T_a \Delta n + n\Delta T_a = n(\delta T_e + \delta T_i) + (T_e + T_i)\delta n
\end{aligned}
\tag{10}
$$

Together with Eqs. (6), Eq. (10) forms an algebraic system of equations relating the jumps δn and δT_α with the total drops Δn and ΔT_α.

The solutions of this system, which are, in general, cumbersome, are considerably simplified if the dimensionless parameter $\lambda k/l$, which figures in (10), is small or large compared with unity. For example, in the case when $\lambda k/l \gg 1$ the solutions have the form

$$\delta n = \Delta n, \quad \delta T_a = \Delta T_a \tag{11}$$

In this case the entire drop in density and temperatures occurs in the region of the mirror. In the opposite limiting case of $\lambda k/l \ll 1$ the changes in n and T_α are distributed more uniformly along the mirror machines, while the jumps at the mirror are

$$\begin{gathered}\delta n = \frac{7}{9}\left(\Delta n + n\frac{\Delta T_e + \Delta T_i}{T_e + T_i}\right), \qquad \delta T_e = \frac{k\varkappa_e}{nl}\left(\frac{\pi m_e}{2T_e}\right)^{1/2}\Delta T_e \\ \delta T_i = \frac{2}{9}\left[\frac{\Delta n}{n}(T_e + T_i) + \Delta T_i + \Delta T_e\right]\end{gathered} \tag{12}$$

By substituting Eqs. (11) or (12) into Eqs. (6) we arrive at the desired expressions for the fluxes of matter and energy for the assigned drops Δn and ΔT_α between the middles of the mirror machines:

$$\begin{aligned} &\lambda k / l \ll 1 \\ &q = -\frac{8}{9k}(2\pi m_i T_i)^{-1/2}[(T_e + T_i)\Delta n + n(\Delta T_e + \Delta T_i)] \\ &Q_e = -\varkappa_e \frac{\Delta T_e}{l} \\ &Q_i = -\frac{20}{9k}\left(\frac{T_i}{2\pi m_i}\right)^{1/2}[(T_e + T_i)\Delta n + n(\Delta T_e + \Delta T_i)] \\ &\lambda k / l \gg 1 \\ &q = -\frac{(2\pi m_i T_i)^{-1/2}}{k}\left[(T_e + T_i)\Delta n + \frac{1}{2}(\Delta T_e + \Delta T_i)\right] \\ &Q_e = -\left(\frac{2T_e}{\pi m_e}\right)^{1/2}\frac{n}{k}\Delta T_e \\ &Q_i = -\frac{1}{k}\left(\frac{T_i}{2\pi m_i}\right)^{1/2}[(T_e + T_i)\Delta n + 3\Delta T_i + \Delta T_e] \end{aligned} \tag{13}$$

The further procedure for the derivation of the equations describing the time and space distribution of the plasma parameters is analogous to that described in [5, 6]. We will confine ourselves to the consideration of one particular case, which is of practical interest, of the evolution of rather long clusters with $L \gg l(m_i/m_e)^{1/4}$, when the temperatures of the electrons and ions are able to become equal to each other and equalized along the magnetic field in the time of expansion:

$$T_e = T_i = T, \quad \frac{\partial T}{\partial s} = 0 \tag{14}$$

Under these conditions the dynamics of the plasma cluster is described by a single equation for the concentration. In order to derive it we write the equation of balance of the number of particles in the segment of a field tube included between the middles of two adjacent mirror machines:

$$\frac{\partial}{\partial t}\left(\frac{1}{l}\int_{-l/2}^{l/2} n\,ds\right) = -\frac{q(l/2) - q(-l/2)}{l} \tag{15}$$

We introduce the value

$$\frac{\partial q}{\partial z} = \frac{1}{l}[q(l/2) - q(-l/2)]$$

with which one can formally operate as with an ordinary derivative. Since the plasma concentration varies little along the length of one mirror machine, one can assume in Eqs. (13) that

$$n = \bar{n} = \frac{1}{l} \int_{-l/2}^{l/2} n\, ds$$

Substituting these equations into (15) and keeping (14) in mind, we obtain the equation for $\bar{n}$:

$$\frac{\partial \bar{n}}{\partial t} = D \frac{\partial^2 \bar{n}}{\partial z^2}, \qquad D = \begin{cases} D_1 = \frac{8l}{9k}\left(\frac{2T}{\pi m_i}\right)^{1/2}, & l_0 \ll \lambda \ll l/k \\ D_2 = \frac{l}{k}\left(\frac{2T}{\pi m_i}\right)^{1/2}, & l/k \ll \lambda \ll lk \end{cases} \tag{16}$$

The equation formulated describes the process of diffusional dispersion of the plasma along the magnetic field. The inequalities pertaining to the coefficient of diffusion are obtained through a combination of conditions (2) and the inequalities $\lambda k/l \ll 1$ and $\lambda k/l \gg 1$ which were used in solving the system of equations (10). For intermediate values of the parameter $\lambda k/l$ the values of the coefficient of diffusion lie in the range of $D_1 < D < D_2$.

By using Eq. (16) one can estimate the velocity of expansion u and the time of longitudinal confinement $\tau \approx L/u$ of the plasma by the corrugated field in the range of mean free paths (2):

$$u \sim v_{Ti} \frac{l}{Lk}, \qquad \tau \sim \frac{L}{v_{Ti}} \frac{Lk}{l}$$

It is assumed that the length of the installation is equal in order of magnitude to the length L of the plasma cluster.

From the estimates presented it is seen that with a transition from a smooth configuration of the field to a multiple-mirror configuration with point mirrors the time of longitudinal confinement of the plasma increases considerably and grows in proportion to the product of the number of mirror machines times their mirror ratio.

At the limits of their applicability with respect to mean free path (2) the equations obtained coincide with the corresponding equations derived in [5, 6] with the accuracy of the numerical coefficient. The results presented are intermediate between the purely kinetic and hydrodynamic modes analyzed in [5, 6], and along with the results of these works they give a complete picture of the dynamics of plasma flow in a strongly corrugated ($k \gg 1$) magnetic field.

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